Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Higher level

Paper 2

21 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.

- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.

- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.

- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.

- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.

- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.

- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.

- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) attempt to use area of triangle formula ***(M1)***

$\frac{1}{2} \times 25.9 \times 6.36 \times \sin(125^\circ)$ ***(A1)***

67.5 m^2 $(67.4700... \text{ m}^2)$ ***A1***

Note: Units are required. The final ***A1*** is only awarded if the correct units are seen in their answer; hence award ***(M1)(A1)A0*** for an unsupported answer of 67.5.

[3 marks]

(b) attempt to use cosine rule ***(M1)***

$(\text{BK} =)\sqrt{12^2 + 6.36^2 - 2 \times 12 \times 6.36 \times \cos 45^\circ}$ ***(A1)***

8.75 (m) $(8.74738\ldots\text{(m)})$ ***A1***

Note: Award ***(M1)(A1)(A0)*** for radian answer of 10.2 (m) $(10.2109\ldots\text{(m)})$ with or without working shown.

[3 marks]

(c) **METHOD 1**

attempt to use sine rule with measurements from triangle OKX ***(M1)***

$\frac{\text{OX}}{\sin 51.1^\circ} = \frac{22.2}{\sin 53.8^\circ}$ ***(A1)***

$(\text{OX} =)\ 21.4 \text{ (m)}$ $(21.4099\ldots)\text{(m)}$ ***A1***

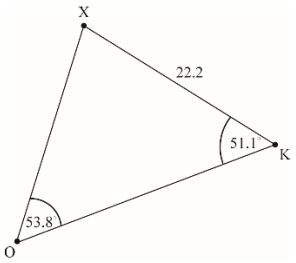
$(21.4 \text{ (m)} < 22.2 \text{ (m)})$

Odette is closer to the football / Khemil is further from the football ***A1***

Note: For the final ***A1*** to be awarded 21.4 $(21.4099\ldots)$ must be seen. Follow through within question part for final ***A1*** for a consistent comparison with their OX.

METHOD 2

sketch of triangle OXK with vertices, angles and lengths ***(A1)***



51.1° is smallest angle in triangle OXK ***R1***

opposite side (OX) is smallest length ***R1***

therefore Odette is closest ***A1***

[4 marks]

continued…

Question 1 continued

(d) attempt to use length of arc formula ***(M1)***

$\frac{135}{360} \times 2\pi \times 12$ ***(A1)***

$28.3\,(\text{m})$ $(9\pi,\ 28.2743...)$ (m) ***A1***

[3 marks]

Total [13 marks]

2. (a) (i) 1200 ***A1***

(ii) the initial population of the bacteria ***A1***

[2 marks]

(b) $1200 \times k^3 = 18750$ ***(A1)***

$(k =)\ 2.5$ ***A1***

[2 marks]

(c) $1200 \times 2.5^{1.5}$ ***(A1)***

$4740\ (4743.41\ldots)$ ***A1***

Note: Do not penalize if final answer is not given as an integer. Award ***(A1)A0*** for an answer of $3950\ (3949.14\ldots)$ from use of 1.3 in the exponent, but only if working is shown.

[2 marks]

(d) equating $P(t)$ and $S(t)$ **OR** equating each function to a common variable ***(M1)***

$1200 \times 2.5^t = 5000 \times 1.65^t$; $1200 \times 2.5^t = x$ and $5000 \times 1.65^t = x$

$t = 3.43$ (hours) $(3.43456...)$ ***A1***

[2 marks]

continued…

Question 2 continued

(e) **METHOD 1**

$5000 \times 1.65^t = 19000$ ***(M1)***

$(t =)\ 2.66586...$ **OR** $(t - 2 =)\ 0.66586...$ (seen) ***(A1)***

multiplying by 60 seen to convert to minutes ***(M1)***
$(m = 39.9521\ldots)$

$(m =)\ 40$ (minutes) **OR** 2 hours and 40 minutes ***A1***

METHOD 2

equating an expression for $S(t)$ to 19000 ***(M1)***

expressing t as $2 + \frac{m}{60}$ ***(A1)***

$5000 \times 1.65^{2+\frac{m}{60}} = 19000$

$2 + \frac{m}{60} = 2.66586\ldots$ ***A1***

$(m =)\ 40$ (minutes) **OR** 2 hours and 40 minutes ***A1***

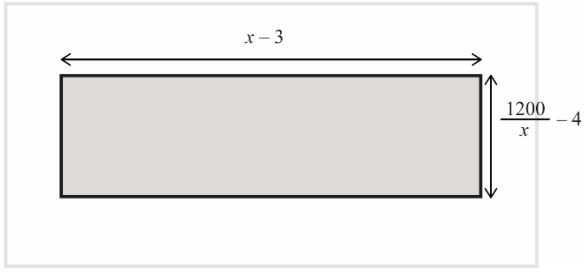
Note: Award ***(M1)(A1)(M1)A0*** for an answer of $39.9521...$ or 39 with or without working.

[4 marks]
Total [12 marks]

3. (a)

Note: In methods 1 and 2, full marks are available for candidates who work with a dummy variable, e.g. y, that represents the width of the park and hence is equal to $\frac{1200}{x}$.
The substitution to express an answer in only x may come as late as the final line.

METHOD 1 (finding dimensions of garden)



(width of park =) $\frac{1200}{x}$ ***(A1)***

(length of garden =) $x-3$, (width of garden =) $\frac{1200}{x}-4$ ***(A1)(A1)***

$A=(x-3)\times\left(\frac{1200}{x}-4\right)$ ***A1***

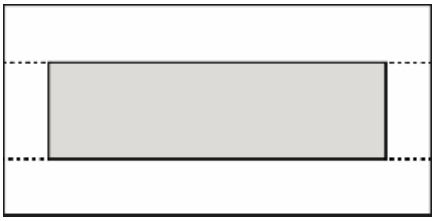
$=1200-4x-\frac{3600}{x}+12$ ***A1***

$=1212-4x-\frac{3600}{x}$ ***AG***

continued…

Question 3 continued

METHOD 2 (subtracting the area of the path)



width of park = $\frac{1200}{x}$ ***(A1)***

attempt to cut path into 4 (or 8) pieces ***(M1)***

four (or eight) areas of the path expressed in terms of x ***(A1)***

$A = 1200 - 2x - 2x - 1.5\left(\frac{1200}{x} - 4\right) - 1.5\left(\frac{1200}{x} - 4\right)$ ***A1***

correct manipulation leading to given result ***A1***

$= 1212 - 4x - \frac{1800}{x} - \frac{1800}{x}$

$= 1212 - 4x - \frac{3600}{x}$ ***AG***

Note: To award ***(M1)(A1)*** without a diagram the division of the park must be clear.

[5 marks]

(b) setting $1212 - 4x - \frac{3600}{x} = 800$ (accept a sketch) ***(M1)***

$x = 9.64$ $(9.64011\ldots)$ (m) **OR** $x = 93.4$ $(93.3598\ldots)$ (m) ***A1***

(width $=$) 124 $(124.479\ldots)$ (m) ***A1***

(width $=$) 12.9 $(12.8534\ldots)$ (m) ***A1***

Note: To award the final ***A1*** both values of x **and** both values of the width must be seen. Accept 12.8 for second value of width from candidate dividing 1200 by 3 sf value of 93.4.

[4 marks]

(c) $\left(\frac{\mathrm{d}A}{\mathrm{d}x} =\right) -4 + \frac{3600}{x^2}$ **OR** $-4 + 3600x^{-2}$ ***A1A1A1***

Note: Award ***A1*** for -4, ***A1*** for $+3600$, and ***A1*** for x^{-2} or x^2 in denominator.

[3 marks]

continued…

Question 3 continued

(d) setting *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ equal to 0 **OR** sketch of *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ with x-intercept highlighted ***M1***

$(x =)\ 30\ (\mathrm{m})$ ***A1***

Note: To award ***A1FT*** the candidate's value of x must be within the domain given in the problem $(3 < x < 300)$.

[2 marks]

(e) **EITHER**

evidence of using GDC to find maximum of graph of $A = 1212 - 4x - \frac{3600}{x}$ ***(M1)***

OR

substitution of *their* x into A ***(M1)***

OR

dividing 1200 by *their* x to find width of park **and** subtracting 3 from *their* x and 4 from the width to find park dimensions ***(M1)***

Note: For the last two methods, only follow through if $3 <$ their $x < 300$.

THEN

$(A =)\ 972\ (\mathrm{m}^2)$ ***A1***

[2 marks]

Total [16 marks]

4. (a) any city can be travelled to or from any other city (so is connected) ***R1***

EITHER

but there is no direct flight between Los Angeles and Dallas (for example) ***R1***

OR

but not every vertex has degree 4 ***R1***

Note: Accept equivalent statements for the cities being connected and the graph not being complete.

[2 marks]

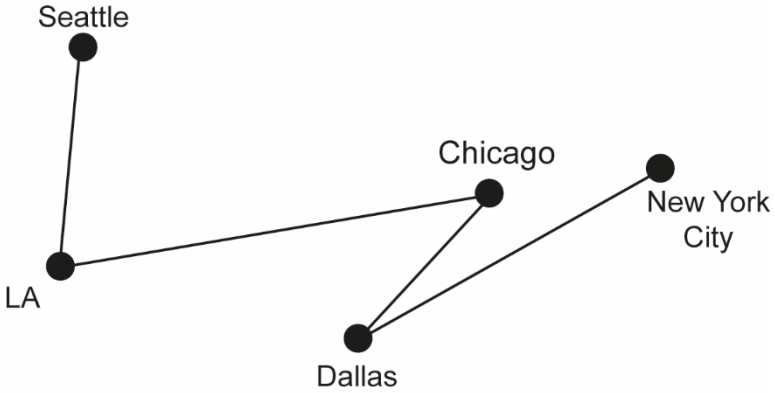
continued…

Question 4 continued

(b) edge CD selected first ***M1***

DN,
CL,
LS ***A1***

Note: Award marks if the answers are written as sums in the correct order.
M1 if 30 is seen first, ***A1*** for $30 + 39 + 41 + 58$.



A1

Note: The final ***A1*** can be awarded independently. Award ***M0A0A1*** for a correct MST graph with no other working. Award ***M1A0A1*** if Prim's algorithm is seen to be used correctly with CD first.

[3 marks]

(c) $2 \times \mathrm{MST}$ weight ***(M1)***

$= \$336$ ***A1***

Note: Allow any integer multiple (>1) of MST weight for ***M1***, and if correctly calculated, award ***M1A1***.

[2 marks]

(d) attempt at nearest neighbour algorithm ***M1***

order is $\mathrm{LA} \to \mathrm{D} \to \mathrm{C} \to \mathrm{NYC} \to \mathrm{S} \to \mathrm{LA}$ ***A1***

Note: Award ***M1*** for a route that begins with LA and then D, this includes seeing 26 as the first value in a sum.
Award ***A1*** if $26+30+68+66+58$ seen in order.

Note: Award ***M1A0*** for an incorrect first nearest neighbour proceeding 'correctly' to the next vertex. For example, LA to C and then C to D.

upper bound is $(26 + 30 + 68 + 66 + 58 =)\ \$248$ ***A1***

Note: Award ***M1A0*** for correct nearest neighbour algorithm starting from a vertex other than LA. Condone the correct tour written backwards i.e. $58 + 66 + 68 + 30 + 26 = 248$

[3 marks]

continued…

Question 4 continued

(e) (i) attempt to find MST of L, N, D and S ***(M1)***
by deleting C, Kruskal gives MST for the remainder as LD, DN, LS
weight 123 ***(A1)***
(lower bound is therefore $123 + (30 + 41) =)$ \$194 ***A1***

Note: Award ***(M1)*** for a graph or list of edges that does not include C.
Award ***(A1)*** if $26 + 39 + 58$ seen in any order.

(ii) by deleting S, Kruskal gives MST for the remainder as LD, DC, DN
weight 95 ***(A1)***
(lower bound is therefore $95 + (58 + 66) =)$ \$219 ***A1***

Note: Award ***(A1)*** if $26 + 30 + 39$ seen in any order.

[5 marks]

(f) $219 \leq C \leq 248$ ***A1A1***

Note: Award ***A1*** for $219 \leq C$ and ***A1*** for $C \leq 248$. Award at most ***A1A0*** for $219 < C < 248$.
FT for their values from part (e) if higher value from (e)(i) and (e)(ii) used for the lower bound, and part (d) for the upper.

[2 marks]

(g) any valid tour, within their interval from part (f), from any starting point **OR**
any valid tour that starts and finishes at N ***(M1)***
valid tour starting point N **AND** within their interval ***A1***
e.g NDCLSN (weight 234)

Note: If part (f) not correct, **only** award ***A1FT*** if their valid tour begins and ends at N **AND** lies within **BOTH** their interval (including if one-sided) in part (f) **AND** $219 \leq C \leq 248$.

If no response in the form of an interval seen in part (f) then award ***M1A0*** for a valid tour beginning and ending at N **AND** within $219 \leq C \leq 248$.

[2 marks]

Total [19 marks]

5.

(a) $(\boldsymbol{T} =) \begin{matrix} \\ \text{(B)} \\ \text{(G)} \\ \text{(N)} \end{matrix} \begin{matrix} \begin{matrix} \text{(B)} & \text{(G)} & \text{(N)} \end{matrix} \\ \begin{pmatrix} 0.945 & 0.015 & 0.02 \\ 0.05 & 0.965 & 0.03 \\ 0.005 & 0.02 & 0.95 \end{pmatrix} \end{matrix}$ ***M1A1A1***

Note: Accept the columns in any order. Accept the transpose of this matrix.

Award ***M1*** for a 3x3 matrix with all values between (but not including) 0 and 1, and all columns (or rows if transposed) adding up to 1, award ***A1*** for one correct row (or column if transposed) and ***A1*** for all rows (or columns if transposed) correct.

[3 marks]

(b) $(\boldsymbol{T}^6 =) \begin{pmatrix} 0.72 & 0.077 & 0.098 \\ 0.24 & 0.83 & 0.16 \\ 0.035 & 0.098 & 0.74 \end{pmatrix}$ ***(M1)***

Note: Accept a transposed matrix.

multiplying their $\boldsymbol{T}^6$ by a correct matrix of the initial populations ***(M1)***

$$\begin{pmatrix} 0.72 & 0.077 & 0.098 \\ 0.24 & 0.83 & 0.16 \\ 0.035 & 0.098 & 0.74 \end{pmatrix} \begin{pmatrix} 26000 \\ 240000 \\ 50000 \end{pmatrix}$$

Note: Award this ***M1*** for a transposed $\boldsymbol{T}$ if used correctly in part (b) i.e. preceded by 1×3 matrix rather than followed by a 3×1 matrix.

$$= \begin{pmatrix} 42133 \\ 212205 \\ 61661 \end{pmatrix}$$ ***(A1)***

so the expected population of the German side would be $212000\ (212205)$ ***A1***

Note: Award ***M0M1A0A1*** for an answer of $174000\ \ (=174031)$. This is the case when $\boldsymbol{T}^{30}$ has been used.

[4 marks]

continued…

Question 5 continued

(c) (i) $\begin{pmatrix} 0.945 & 0.015 & 0.02 \\ 0.05 & 0.965 & 0.03 \\ 0.005 & 0.02 & 0.95 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$

at least two of these three:

$0.945u_1 + 0.015u_2 + 0.02u_3 = u_1$

$0.05u_1 + 0.965u_2 + 0.03u_3 = u_2$ ***A1***

$0.005u_1 + 0.02u_2 + 0.95u_3 = u_3$

and

$u_1 + u_2 + u_3 = 1$ (may be seen in part (c)(ii)) ***A1***

(ii) $(\boldsymbol{u} =) \begin{pmatrix} 0.231 \\ 0.533 \\ 0.236 \end{pmatrix}$ $\left(\boldsymbol{u} = \begin{pmatrix} 0.231155... \\ 0.532663... \\ 0.236180... \end{pmatrix} \right)$ ***A1***

Note: The ***A1*** in part (c)(ii) can be awarded independently of the working in part (c)(i).

[3 marks]

(d) $0.532663... \times (26000 + 240000 + 50000)$ ***(M1)***

$= 168000$ $(168321...)$ ***A1***

Note: Award ***(M1)A1*** for answers using $\boldsymbol{T}^n$ with n large that lead to a correct answer.
Award ***(M0)A0*** for answers that use $\boldsymbol{T}^n$ that lead to an incorrect answer.

[2 marks]

(e) *Award **R1** for each appropriate reason. For example:*

Movement unlikely to be constant

Total population for entire region likely to grow over time

Each power of the transition matrix takes five years; a relatively long time in terms of population movement.

There may be other/new external factors such as wars in other adjoining countries, leading to an influx of economic migrants.

R1R1

Note: Do not award ***R1*** for any response that shows a lack of understanding of the assumption that the total population remains constant.

[2 marks]

Total [14 marks]

6. (a) slugs appear discretely / independently / randomly / at a constant (average) rate / mean is (approximately) equal to variance ***R1R1***

[2 marks]

(b) new $(m=)\ 0.2\times12\ (=2.4)$ (so $X \sim \text{Po}(2.4)$) ***(A1)***

attempt to use a pdf (e.g $\text{P}(X=4)$) ***(M1)***

$0.125\ (0.125408...)$ ***A1***

[3 marks]

(c) $\text{P}(X<3)$ **OR** $\text{P}(X\leq 2)$ ***(A1)***

$0.570\ (0.569708...)$ ***A1***

[2 marks]

(d) $\text{P}(X\geq 1)=0.909282...$ ***(A1)***

raising a probability to a power of 3 ***(M1)***

$0.909282...^3$

$=0.752\ (0.751788...)$ ***A1***

Note: Award at most ***(A1)(M1)(A0)*** for a final answer of 0.751. Working may not be seen.

[3 marks]

(e) $\text{H}_0 : m=2.4$, ***A1***

$\text{H}_1 : m>2.4$ ***A1***

Note: The hypotheses may be written in words but must include reference to the mean (e.g. "number of snails" is not sufficient to award ***A1***), and state clearly for H_1 that the mean increases.

[2 marks]

(f) **EITHER**

finding either $\text{P}(X\geq 7)$ or $\text{P}(X\geq 8)$ ***(M1)***

$\left(\text{P}(X\geq 7)=\right)\ 0.01160...$ **AND** $\left(\text{P}(X\geq 8)=\right)\ 0.00334...$ ***A1***

OR

finding either $\text{P}(X\leq 7)$ or $\text{P}(X\leq 6)$ ***(M1)***

$\left(\text{P}(X\leq 7)=\right)\ 0.996661...$ **AND** $\left(\text{P}(X\leq 6)=\right)\ 0.988405...$ ***A1***

THEN

so critical region is $X\geq 8$ **OR** $X>7$ ***A1***

Note: ***(M1)A0A1*** can be awarded for a correct answer that is unsupported.

[3 marks]

(g) $(0.75\times12=)\ 9$ ***(A1)***

$\text{P}\left(X\leq 7 \mid m=9\right)$ ***(M1)***

$=0.324$ ***A1***

[3 marks]

Total [18 marks]

7. (a) $\begin{vmatrix} -4-\lambda & 6 \\ 9 & -1-\lambda \end{vmatrix} = 0$ ***(M1)***

Note: Do not accept $\det(\boldsymbol{A}-\lambda\boldsymbol{I})=0$ or similar as evidence of a correct method unless A is explicitly defined to be the given matrix.

$(-4-\lambda)(-1-\lambda)-54=0$

$\lambda=-10,\ \lambda=5$ ***A1A1***

For $\lambda=-10$

$\begin{pmatrix} -4 & 6 \\ 9 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -10x \\ -10y \end{pmatrix}$ ***(M1)***

$-4x+6y=-10x$

$x+y=0$

possible eigenvector is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ (or equivalent) ***A1***

for $\lambda=5$

$\begin{pmatrix} -4 & 6 \\ 9 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 5x \\ 5y \end{pmatrix}$

$-4x+6y=5x$

$3x=2y$

possible eigenvector is $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (or equivalent) ***A1***

Note: If both eigenvalues are incorrect then award at most ***M1A0A0M1A0A0***.

[6 marks]

(b) attempt to substitute their eigenvalues and eigenvectors equation ***(M1)***

$\begin{pmatrix} x \\ y \end{pmatrix}=A\mathrm{e}^{-10t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}+B\mathrm{e}^{5t}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ***A1***

Note: Award at most ***(M1)A0*** if $\begin{pmatrix} x \\ y \end{pmatrix}$ not seen.

[2 marks]

continued…

Question 7 continued

(c) At $t = 0$, $x = 500$ and $y = 125$

$x = -A + 2B$ and $y = A + 3B$

Solving simultaneously: ***(M1)***

$A = -250$ and $B = 125$ ***A1***

$$\left(\begin{pmatrix} x \\ y \end{pmatrix} = -250\mathrm{e}^{-10t}\begin{pmatrix} -1 \\ 1 \end{pmatrix} + 125\mathrm{e}^{5t}\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right)$$

Note: Follow through from their eigenvectors.
Accept equivalent values for A and B based on the direction of their eigenvectors and the order of their eigenvalues in the equation.

[2 marks]

(d) $2:3$ ***A1***

[1 mark]

(e) attempt to eliminate $\mathrm{d}t$ from the two differential equations ***M1***

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9x - y}{-4x + 6y}$$

substituting initial conditions ***(M1)***

$$= \frac{9(500) - 125}{-4(500) + 6(125)}$$

$= -3.5$ ***A1***

Note: Award ***M1*** for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4x + 6y}{9x - y}$.

[3 marks]

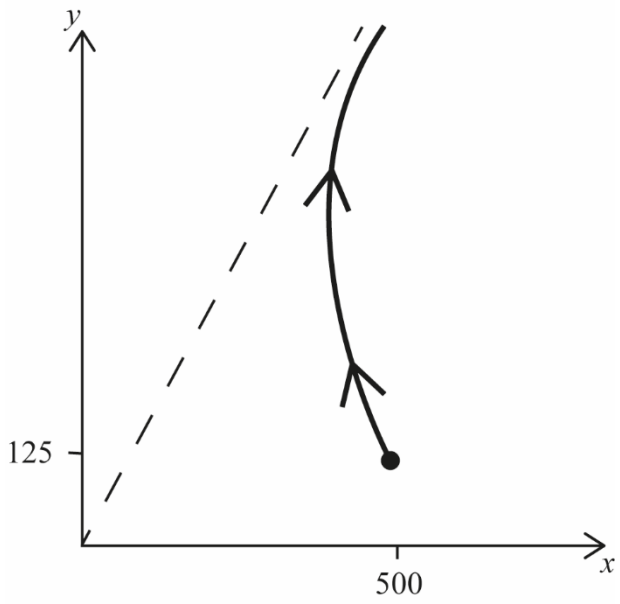
continued…

Question 7 continued

(f) trajectory or trajectories that are consistent with their eigenvalues ***A1***
a trajectory that passes through the point $(500, 125)$ with gradient that is consistent with the response to part (e) ***A1***
the diagram contains at least one of their eigenvectors ***A1***
(e.g. labelled $y = 1.5x$; $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\lambda = 5$ etc.)
the trajectory that passes through $(125, 500)$ tends towards an oblique asymptote that corresponds to their eigenvector and the direction is indicated by at least one arrow on the trajectory ***A1***

Note: For the second ***A1***, the point $(500, 125)$ may not be labelled but there should be a point marked on the trajectory that is consistent with these coordinates.

The final ***A1*** will depend on their eigenvalues. Follow through can be awarded as long as the direction of the trajectory is consistent with the nature of their eigenvalues and eigenvectors.



[4 marks]
Total [18 marks]